Formulating Optimization Objectives: Cardinal Utility Models

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Optimization & Learning Approaches to Resource Allocation for Social Good

Defining Objectives: Background

- There's a whole field (welfare economics) that studies how to use microeconomics to analyze well-being at the aggregate level
- The concerns have tended to be more global (policy impacts, taxation, etc)
- We're largely concerned here with allocation by *local institutions*
- Market designers often think about these issues
  - Typically with ordinal preferences
  - Residency match (Roth, *J. Pol. Econ. 1984*), labor market matching more generally (Roth and Xing, *AER, 1994*), and public school choice (Abdulkadiroglu et al, *AER 2005*) are good examples
  - Different evaluation criteria, e.g. stability, truthfulness, percentage receiving first choice, effects on naive and sophisticated players, etc.
  - Avoids interpersonal utility comparisons

Defining Objectives: Cardinal Utilities

- In many cases we care about levels of outcomes, not just ranked preferences (Anshelevich and Das, *SIGecom Exchanges 2010*). Examples:
  - Homelessness: Percentage stably housed n years after exit from homeless services, cost to relevant social service systems (medical, criminal justice, etc), life outcomes of children (Kube, Das, and Fowler, *AAAI 2019*, Azizi et al, *CPAIOR 2018*)
  - Social services more generally: Kids going to college, lifetime incomes, etc. (Katz et al, *QJE 2001*, Chetty et al, *QJE 2011*).
  - Organ donation: Quality-Adjusted Life Years (QALY) (Zenios, *Man. Sci. 2002*), expected graft survival time (Li et al, *EC 2019*), waiting time until transplant, cost
  - Pair programming: Productivity (Dawande et al, *Man. Sci. 2008*)

Other Use Cases For Cardinal Utilities

- Individuals involved in allocation or matching problems need to reason about levels for decision-making
- Fundamentally, assess $\Pr(A)V(A) \gtreqqless \Pr(B)V(B)$
  - In the Boston Mechanism for School Choice, should I rank my second-most preferred school higher than my most preferred? (Ergin and Sönmez, *J. Pub. Econ 2006*)
- Sometimes there's an explore-exploit tradeoff:
  - Should I ask Stan out on a date, or Mike? (Das and Kamenica, *IJCAI 2005*
  - Should we invite John for a faculty interview, or Bryan? (Das and Li, *WINE 2014*; cf. Lee and Schwartz *RAND J. Econ 2017*)

Defining Objectives: Social Welfare

- Intrinsically normative question. Commonly proposed answers for modeling:
  - Utilitarian: Additive over all agents
  - Rawlsian: Max-Min
  - Nash Bargaining: Multiplicative
- Could also do constrained optimization: e.g. utilitarian subject to some fairness constraints
- Useful to examine case studies on how to frame the objectives and the optimization.

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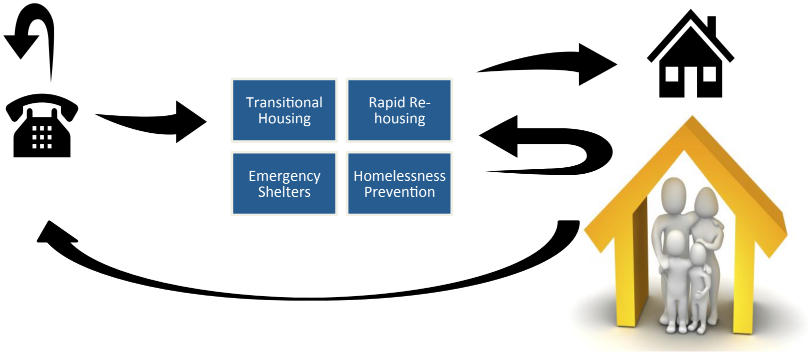
Case Study #1: Homelessness in the US

- Endemic and costly public health threat.
- One night in January 2014: $> 1/2$ million Americans experiencing homelessness
  - Majority in emergency shelters or temporary accommodations
- Very costly ($>$\$4000 / month per individual between health care, criminal justice and homeless services)
- Homelessness is often a "base" problem – hard to deliver other services (mental health, etc) effectively to homeless population

The Homeless System

- Largely funded by HUD
- Network of local agencies
  - In 2014: 23,587 agencies across 416 communities
- Federally required universal elements plus local discretion in service delivery
- Running emergency shelters, providing access to short- and long-term housing support

Homelessness Services



How Do We Assess Efficacy?

- HUD: Are households *safely and stably housed* some time after leaving homeless services?
- Many other criteria: Contacts with Medicaid, Child Protective Services, Criminal Justice system?
  - Danger of criminalizing poverty (Eubanks, 2018)
  - Use of public services is recorded, and becomes available for algorithmic analyses, while those with money can pay for private services
- Expected re-entries: Measure probability of re-entry within two years of exit for each household (Kube, Das, and Fowler *AAAI 2019*)

Optimal Allocation

Optimization Problem

$$
\begin{aligned}
\min_{x_{ij}} \sum_i \sum_j p_{ij} x_{ij} & \\
\text{subject to} \quad \sum_j x_{ij} = 1 & \quad \forall i \\
\sum_i x_{ij} \leq C_j & \quad \forall j
\end{aligned}
$$

- x_{ij}: whether or not household i is placed in intervention j
- p_{ij}: probability of household i reentering if they are placed in intervention j
- C_j: capacity of intervention j

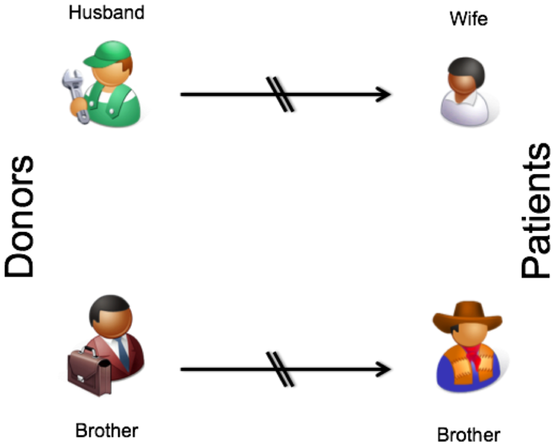
Fairness Constraints

- Allocations may be because of policy constraints
  - E.g. require prioritization of those fleeing domestic abuse
- We can require the allocation to not hurt anyone more than a small percentage in expectation
- Add a constraint
  - $$\sum_j p_{ij} x_{ij} \leq \sum_j p_{ij} y_{ij} + 0.05 \; \forall i$$
  - y_{ij} represents whether or not household i was originally placed in intervention j

Case Study #2: Living Donor Kidney Transplantation

- About $100,000$ people waiting for kidney transplants in the US (2016)
- About, $19,500$ kidney transplants in recent years, ~ 5500 from living donors
- Unfortunately, willing living donors are often not medically compatible.
- One option for them is to enter a *kidney exchange* program (Roth, Sönmez, and Ünver, *QJE 2004*, Abraham, Blum, and Sandholm, *EC 2007*, Dickerson et al *EC 2016*)

Kidney Exchange



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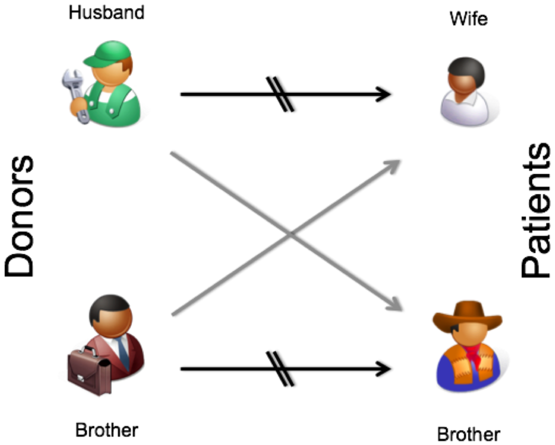
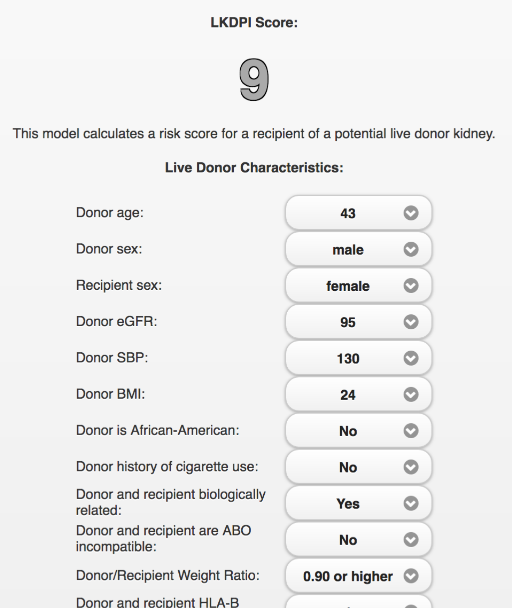


Kidney Exchange: Optimization Objectives

- Usually algorithms try to maximize the number of transplants .
- Sometimes this is done on a weighted graph that takes into account different things (like probability of failure), and requires weighted matching algorithms (e.g. Dickerson, Procaccia, and Sandholm, *AAAI 2012*)
- However, doesn't actually take into account quality of each proposed transplant
  - Conventional wisdom: *Any* living donor transplant is better than any cadaveric transplant, so they're all definitely good enough.

Measuring Match Quality

LKDPI introduced by Massie et al (*Am. J. Transplantation, 2016*)



LKDPI Score:

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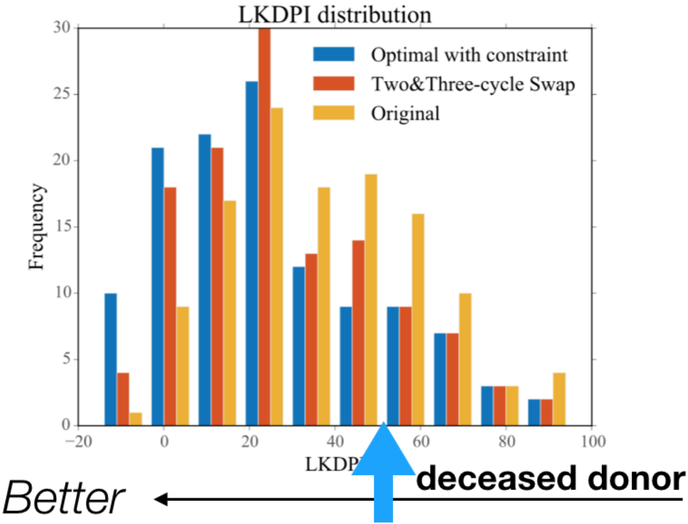
This model calculates a risk score for a recipient of a potential live donor kidney.

Live Donor Characteristics:

Donor age:	43
Donor sex:	male
Recipient sex:	female
Donor eGFR:	95
Donor SBP:	130
Donor BMI:	24
Donor is African-American:	No
Donor history of cigarette use:	No
Donor and recipient biologically related:	Yes
Donor and recipient are ABO incompatible:	No
Donor/Recipient Weight Ratio:	0.90 or higher
Donor and recipient HLA-B	

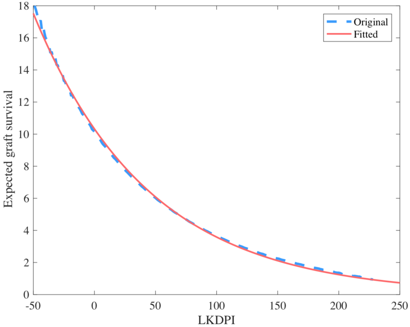
Single Center Analysis

- De-identified data from 2014 - 2016 (Li et al, *EC 2019*)
  - All donor and recipient characteristics for calculating LKDPI

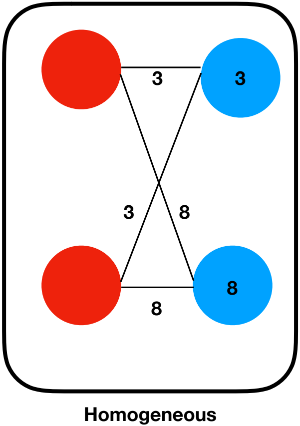


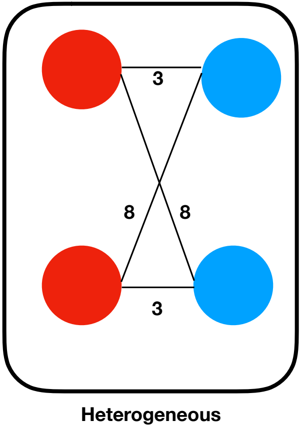
From LKDPI to Graft Survival

- Expected graft survival: estimated as a function of LKDPI $14.78e^{-0.01239LKDPI}$



Heterogeneity of Match Quality





Heterogeneity of Match Quality: The Data

	LKDPI original	LKDPI 2&3 swap	LKDPI Optimal
Original dataset	37.15	25.50	22.46

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Takeaway: Largely donor driven, but with some pairwise idiosyncracies

Incorporating Match Quality in Optimization

- We've built a simulator that we will release to generate
- Can be incorporated in many different kinds of optimization (static, dynamic, hybrid, different optimization goals)
- Will talk more about one model later in this tutorial